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Transient Behavior of Viscoelastic Fluid in an Extruder

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Synopsis

A simplified model is used for calculating the time-dependent velocity of polymeric fluid in an extruder. The flow properties of the fluid are characterized by a simple constitutive equation based on two parameters: a constant viscosity μ and a constant elasticity modulus G. It was found that the transient velocity fluctuates periodically, and the time t_t needed to restore the steady-state velocity from a disturbance varies with the ratio G/μ and the dimensionless group $\rho H^2 G/\mu^2$, where ρ is the density of the fluid and H is the screw depth of the extruder.

Introduction

The starting or sudden perturbation of an extrusion process sometimes involves a fluctuation of volume flow rate during the transient period, which results in a wasting of materials. Thus, the transient period t_t for restoring steady-state velocity after a disturbance should be as short as possible. In this work the effect of fluid properties on t_t was investigated.

Mathematical Model

McKelvey[1] has proposed a simplified flow geometry for simulating the flow patterns observed in the channel along an extruder screw. This model is illustrated in Figures 1 and 2. Flow is induced in the channel (Fig. 2) by the movement of the channel wall at $x_2 = H$ in the direction indicated by the vector V at an angle θ to the x_1 axis. To simplify the problem and still retain the essential features of the flow under investigation, we have made the following assumptions:

(*1*) Molten material transported as a continuous medium in the extruder.

(*2*) No leakage through the radial clearance between the flight and the inside barrel surface.

(*3*) Incompressible fluid.

(*4*) Negligible viscous dissipation effects; i.e., flow assumed to be isothermal.

(*5*) The angle θ is zero, and the components of the velocity are assumed to be:

$$v_1 = v_1(x_2, x_3\, t)$$
$$v_2 = v_3 = 0$$

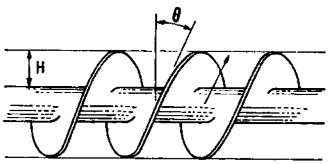

Fig. 1. Geometry of extruder screw.

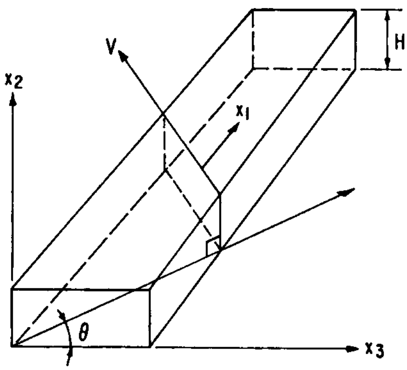

Fig. 2. Coordinate axes used in derivation of the field equations.

(*6*) Flow field infinite in extent in the x_1 direction (this implies no velocity gradients or shear stress gradients in the x_1 direction)

The continuity equation states that

$$\partial v_1/\partial x_1 + \partial v_2/\partial x_2 + \partial v_3/\partial x_3 = 0 \tag{1}$$

and we see that the assumed velocity distribution in assumption (*5*) above satisfies eq. (1) perfectly.

The equations of motion are (neglecting the body forces):

$$-\partial p/\partial x_1 + \partial p_{12}/\partial x_2 + \partial p_{13}/\partial x_3 = \rho(\partial v_1/\partial t) \tag{2}$$

$$-\partial p/\partial x_2 + \partial p_{22}/\partial x_2 + \partial p_{23}/\partial x_3 = 0 \tag{3}$$

$$-\partial p/\partial x_3 + \partial p_{33}/\partial x_3 + \partial p_{23}/\partial x_2 = 0 \tag{4}$$

where p is the hydrostatic pressure, and p_{ij} are the components of the shear stress tensor. It should be noted here that p is a scalar variable to be determined, together with the velocity components v_1, v_2, and v_3, from eqs. (1)–(4) after a suitable constitutive equation for p_{ij} has been selected. The following analysis reveals that p is not a function of time.

For the sake of retaining the viscoelastic nature of the medium and yet keeping things simple enough to be manageable we decided to use the White-Metzner[2] constitutive equation. This equation, expressed in cartesian tensor form, is

$$2Ge_{ij}^* - p_{ij}/\lambda = \partial p_{ij}/\partial t + v_k(\partial p_{ij}/\partial x_k) - p_{ik}(\partial v_j/\partial x_k) - p_{kj}(\partial v_i/\partial x_k) \tag{5}$$

where $2e_{ij}^* = \partial v_j/\partial x_i + \partial v_j/\partial x_i$, G is the modulus of elasticity (a constant), and λ is the relaxation time, a function of the invariants of the stress tensor. To simplify the analysis further, we have assumed the relaxation time to be constant, which is equivalent to assuming a constant viscosity, since $\mu = \lambda G$.

Equations for each of the six independent components of the shear stress tensor can be written from eq. (5) as follows:

$$-p_{11}/\lambda = \partial p_{11}/\partial t - 2p_{12}(\partial v_1/\partial x_2) - 2p_{13}(\partial v_1/\partial x_3) \tag{6}$$

$$-p_{22}/\lambda = \partial p_{22}/\partial t \tag{7}$$

$$-p_{33}/\lambda = \partial p_{33}/\partial t \tag{8}$$

$$G(\partial v_1/\partial x_2) - p_{12}/\lambda = \partial p_{12}/\partial t - p_{22}(\partial v_1/\partial x_2) - p_{23}(\partial v_1/\partial x_3) \tag{9}$$

$$G(\partial v_1/\partial x_3) - p_{13}/\lambda = \partial p_{13}/\partial t - p_{23}(\partial v_1/\partial x_2) - p_{33}(\partial v_1/\partial x_3) \tag{10}$$

$$-p_{23}/\lambda = \partial p_{23}/\partial t \tag{11}$$

If we consider an unsteady motion starting with the fluid in a completely relaxed state, all p_{ij} are initially zero. Thus, the only values for p_{22}, p_{33}, and p_{23} that will satisfy eqs. (7), (8), and (11) are

$$p_{22} = p_{33} = p_{23} = 0 \tag{12}$$

Substituting the relations (12) into eqs. (3) and (4), one can show that p is a function of x_1 only. Differentiating eq. (2) with respect to x_1 shows that $\partial p/\partial x_1$ is a constant. Hereinafter we shall consider that the pressure gradient $\partial p/\partial x_1$ is zero, which means that the flow results only from the movement of the channel wall at $x_2 = H$.

By combining eqs. (2), (9), and (10) one obtains

$$(1/\mu)(\partial v_1/\partial t) + (1/G)(\partial^2 v_1/\partial t^2) = (1/p)(\partial^2 v_1/\partial x_2^2 + \partial^2 v_1/\partial x_3^2) \tag{13}$$

Equation (13) together with the appropriate boundary conditions defines the velocity in the channel as a function of time and position. This velocity distribution can be combined, in principle, with eqs. (6), (9), and (10) for a calculation of the stress distribution in the channel. Therefore, the assumed velocity distribution, assumption (*5*), is valid, and no secondary flow will occur.

The problem can be further simplified, without alteration of the qualitative results sought, by considering the channel to be of infinite width. For this case velocity gradients in the x_3 direction vanish, $v_1 = f(x_2, t)$, and eq. (13) simplifies to

$$(1/\mu)(\partial v_1/\partial t) + (1/G)(\partial^2 v_1/\partial t^2) = (1/p)(\partial^2 v_1/\partial x_2^2) \tag{14}$$

The boundary conditions for the case in which the channel wall at $x_2 = H$ is suddenly accelerated from rest to a constant velocity V are:

$$\begin{aligned} &\text{At } t = 0 \text{ for all } x_2: && v_1 = 0 \text{ and } \partial v_1/\partial t = 0 \\ &\text{At } t > 0 \text{ and } x_2 = 0: && v_1 = 0 \\ &\text{At } t > 0 \text{ and } x_2 = H: && v_1 = V \end{aligned} \tag{15}$$

Equation (14) with the boundary conditions (15) has been solved analytically, and the solution has the form

$$v_1 = Vx_2/H + V(A_n + B_n) \tag{16}$$

where Vx_2/H represents the steady-state velocity profile, and $V(A_n + B_n)$ represents the transient velocity profile:

$$A_n = -\sum_{n=1}^{N}(-1)^{n+1}2n\pi \sin(n\pi x_2/H)\{[\exp\{(-1+\alpha_n)(Gt/2\mu)\}]/[2n^2\pi^2 - (\rho H^2G/2\mu^2)(1-\alpha_n)] + [\exp\{(-1-\alpha_n)(Gt/2\mu)\}]/[2n^2\pi^2 - (\rho H^2G/2\mu^2)(1-\alpha_n)]\} \tag{17}$$

$$B_n = -\sum_{n=N+1}^{\infty}(-1)^{n+1}4n\pi \sin(n\pi x_2/H) \times \left\{\frac{[(pH^2/\mu)\beta_n \sin\beta_n t - \{(pGH^2/2\mu^2) - 2n^2\pi^2\}\cos\beta_n t]}{[(\rho GH^2/2\mu^2) - 2n^2\pi^2]^2 + [(\rho H^2/\mu)\beta_n]^2}\right\}\exp\{Gt/2\mu\} \tag{18}$$

where

$$\alpha_n = (1 - 4n^2\pi^2\mu^2/\rho GH^2)^{1/2}$$

$$\beta_n = (G/\mu)[(n^2\pi^2\mu^2/\rho GH^2) - 1/4]^{1/2}$$

and N is a positive integer, such that

$$4N^2\pi^2\mu^2/\rho GH^2 < 1 > 4(N+1)^2N^2\mu^2/\rho GH^2$$

For ordinary polymers $4n^2\pi^2\mu^2/\rho GH^2 > 1$; therefore, only B_n determines the transient velocities.

TABLE I
Effect of G, μ, and H on t_t

G	μ	H	t_t, sec.
10[a]	10,000	3	12,950(±25)
1,000[a]	10,000	3	110(±10)
10,000[a]	10,000	3	30(±10)
10	100[b]	3	250(±50)
10	1,000[b]	3	1,100(±50)
10	10,000[b]	3	12,950(±50)
10	100,000[b]	3	49,950(±50)
10	10,000	1[c]	15,150(±50)
10	10,000	2[c]	16,250(±50)
10	10,000	3[c]	12,950(±50)
10	10,000	4[c]	14,050(±50)
10	10,000	5[c]	16,500(±50)

[a] Effect of G on t_t.
[b] Effect of μ on t_t.
[c] Effect of H on t_t.

Discussion of Results from Calculation

The velocity of viscoelastic fluid in an extruder fluctuates in a very short period (fraction of a second) during the transient period (see Fig. 3). This

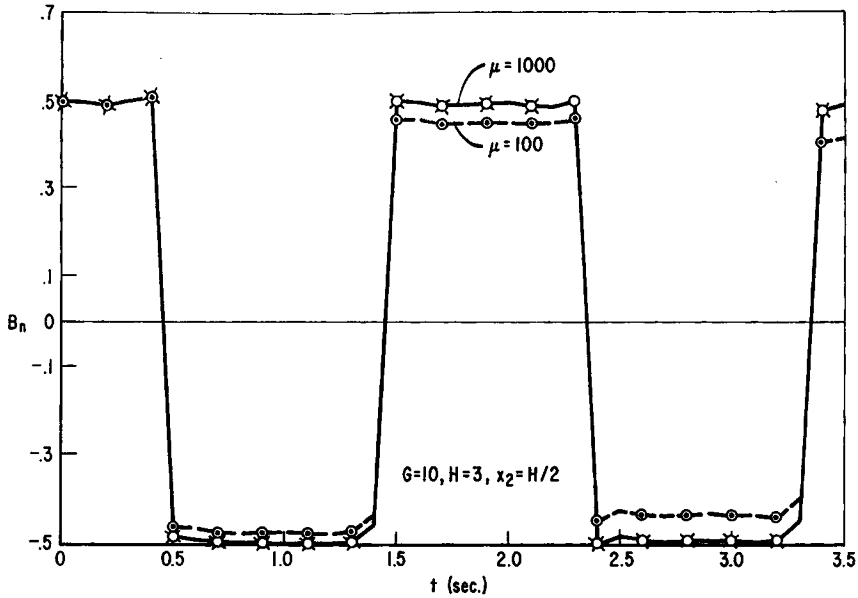

Fig. 3. B_n (deviation from steady-state velocity) versus time.

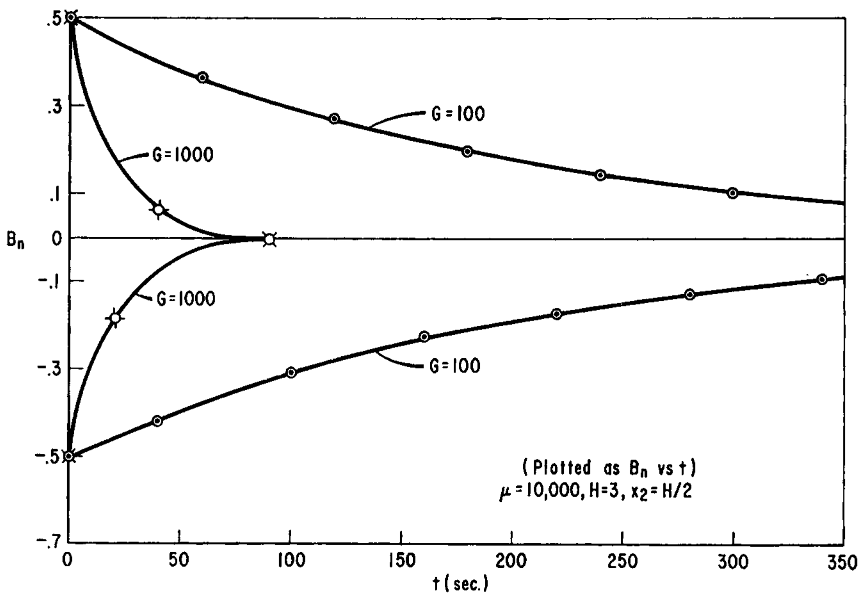

Fig. 4. Effect of modulus of elasticity on transient velocity.

qualitative phenomenon has been recently verified, experimentally, by Hermes and Fredrickson.[3] The amplitude of fluctuation decreases with increase of G and decrease of μ (see Figs. 4 and 5), whereas it is insensitive to the change of H (see Fig. 6). By examining the figures we know that

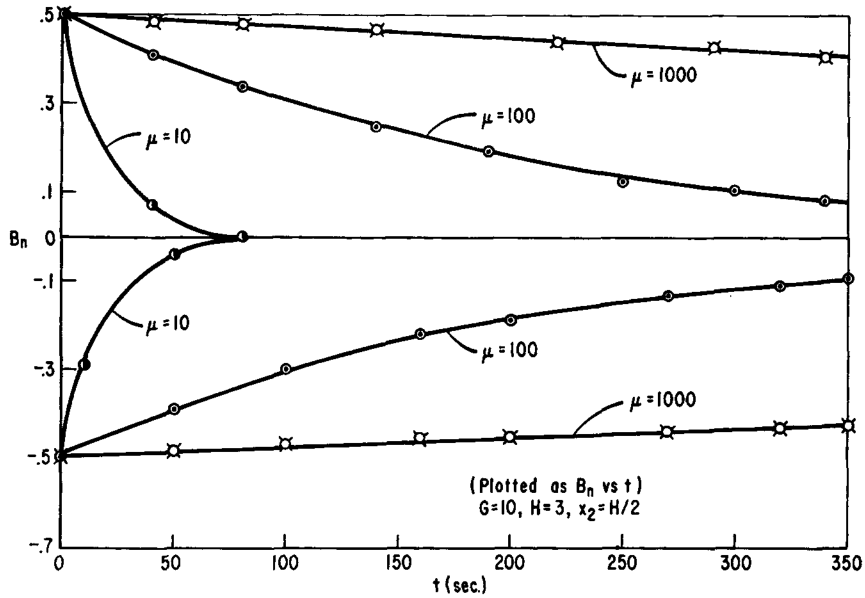

Fig. 5. Effect of apparent viscosity on transient velocity.

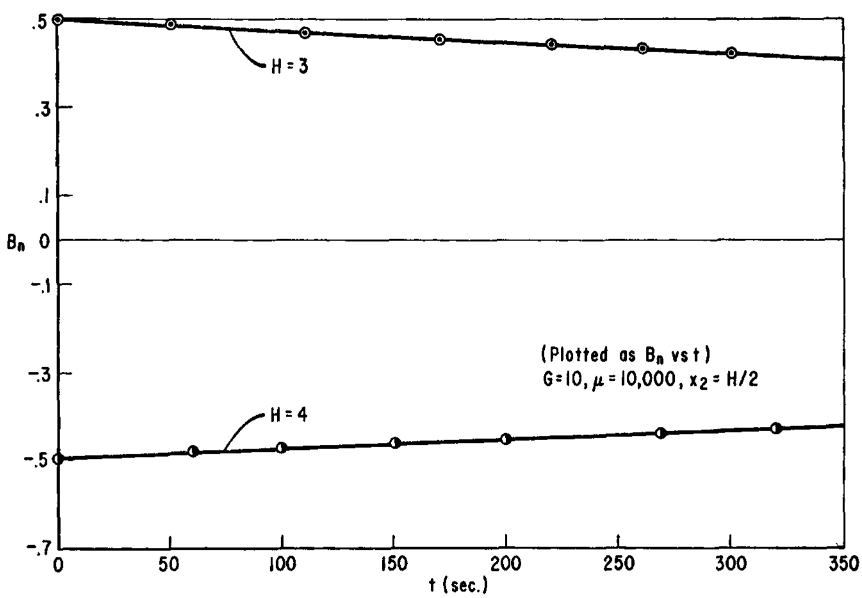

Fig. 6. Effect of depth of extruder channel on transient velocity.

the length of the transient time t_t varies in the same fashion as the amplitude of fluctuation. This is demonstrated in Table I.

Provided experimental correlations between molecular properties of the polymeric fluid (such as molecular weight and molecular weight distribution) and the parameters G and μ have been established, one can predict the effect of the molecular properties on the amplitude of volume rate fluctuation and the transient period t_t.

Conclusion

The transient volume rate of viscoelastic fluid in an extruder fluctuates in a very short period. The amplitude of fluctuation and the transient time t_t (needed to restore the steady-state velocity from a disturbance) decrease with the increase of elasticity and with the decrease of viscosity of the fluid, but they are insensitive to the change of depth of the extruder channel.

Nomenclature

e_{ij}^* = Component of rate of strain tensor [see eq. (5)], sec.$^{-1}$
G = Modulus of elasticity, g./cm.-sec.^{2}
H = Height of extruder channel, cm.
p = Hydrostatic pressure, g./cm.-sec.^{2}
p_{ij} = Component of shear stress tensor [see eq. (5)], g./cm.-sec.^{2}
t = Time, sec.
t_t = Transient time for restoring steady-state velocity after a disturbance has been imposed, sec.
V_i = Velocity component, cm./sec.
V = Velocity of channel wall at $x_2 = H$, cm./sec.
W = Width of extruder channel, cm.
x_i = Cartesian coordinate, cm.
λ = Relaxation time [see eq. (5)], sec.
μ = Apparent viscosity, g./cm.^{3}

Gratitude is extended to J. Vrentas, L. Johns, and L. Duda, of The Dow Chemical Company, for many illuminating discussions.

References

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Résumé

Un modèle simplifié est utilisé en vue de calculer la vitesse d'un fluide polymérique dans un extrudeur en fonction du temps. Les propriétés d'écoulement du fluide sont caractérisées par une équation de caractérisation simple basée sur deux paramètres: une viscosité constante μ, et un module constant d'élasticité G. On a trouvé que la vitesse de transition fluctuait périodiquement et le temps t_t nécessaire à restaurer la vitesse stationnaire au départ de la perturbation varie avec le rapport G/μ et le groupe sans dimensions $\rho H^2G/\mu^2$, où ρ est la densité du fluide et H, la profondeur de la vis de l'extrudeur.

Zusammenfassung

Mittels eines vereinfachten Modells wird die zeitabhängige Geschwindigkeit einer polymeren Flüssigkeit in einem Extruder berechnet. Die Fliesseigenschfaten der Flüssigkeit werden durch eine enfache fundamentale Gleichung charakterisiert, die auf zwei Parametern basiert: einer konstanten Viskosität μ und einem konstanten Elastizitätsmodul G. Es wurde gefunden, dass die augenblickliche Geschwindigkeit periodisch schwankt, und dass die Zeit t_t, die nötig ist, um nach einer Störung die stationäre Geschwindigkeit wiederherzustellen, vom Verhältnis G/μ und vom dimensionslosen Ausdruck $\rho H^2 G/\mu^2$ abhängt, wobei ρ die Dichte der Flüssigkeit und H die Schraubenganghöhe des Extruders ist.

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